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Arthur Prior and 'Now'

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Abstract On the 4th of December 1967, Hans Kamp sent his UCLA seminar notes on the logic of 'now' to Arthur N. Prior. Kamp's two-dimensional analysis stimulated Prior to an intense burst of creativity in which he sought to integrate Kamp's work into tense logic using a one-dimensional approach. Prior's search led him through the work of Castañeda, and back to his own work on hybrid logic: the first made temporal reference philosophically respectable, the second made it technically feasible in a modal framework. With the aid of hybrid logic, Prior built a bridge from a two-dimensional UT calculus to a one-dimensional tense logic containing the 'now' operator J. Drawing on material from the Prior archive, and the paper "'Now'" that detailed Priors findings, we retell this story. We focus on Prior's completeness conjecture for the hybrid system and the role played by temporal reference.

Keywords Arthur Prior · Hans Kamp · two-dimensional logic · hybrid logic · now

1 Introduction

In the fall of 1967, Richard Montague held a seminar on pragmatics at UCLA. At that seminar Hans Kamp, still a PhD-student, introduced what today is known as two-dimensional logic and used it to analyze the temporal indexical 'now'.[1] On the 4th of December of the same year, Kamp sent his seminar notes to Arthur N. Prior, and they inspired Prior profoundly. In response to Kamp's notes, Prior worked intensively

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[1] A copy of the notes Kamp used for his presentation can be found in the Prior Archive at the Bodleian Library, Oxford (Kamp 1967).

for the rest of December, striving to understand Kamp's work and its ramifications, and developing an alternative. During that month he sent eight letters back to Kamp. The result of Prior's activity was published as "'Now'" in *Noûs* (Prior 1968).[2] Kamp, however, didn't publish on the subject until "Formal properties of 'now'" (1971), the paper which is usually considered to be the starting point for the technical study of the logic of indexicals, appeared in *Theoria*. Now, Kamp's methods, especially his use of two-dimensional semantics, have been highly influential; but the interaction between Kamp and Prior four years previously should not be overlooked. As we shall see, Prior's letters and the paper "'Now'" provide a sophisticated discussion and a detailed response to Kamp's work. They remain relevant to contemporary concerns and, remarkably, seem to be the only example of Prior using hybrid logic to analyze temporal reference in natural language.

The letters Prior sent are essentially technical: how can we formalise an indexical notion like 'now' at all? But from the published paper we see that Prior does not see the problem as merely technical. On the contrary, he addresses the many philosophical and linguistic issues that 'now' raises. So his response is typically Priorean in that (as so often before) he tries to create "a dialogue between the constructors of formalisms and the recorders of idioms" (Prior 1968/2003, p. 173).

Prior's linguistic and philosophical commentary drew on both English usage and on Castañeda's (1967) analysis of indexicals like 'I', 'here' and 'now'. His technical analysis moved through two versions of his UT calculus, made a brief but crucial intermezzo into hybrid logic, and concluded with a detailed study of tense logic enriched with what today is known as the universal modality $\Box$ and a Kamp-style 'now' operator J. As this summary suggests, Prior's discussion is both interesting and demanding; the paper ranges widely, and makes many detailed points. This paper focuses on two, which we regard as central. First, Prior demonstrates that Kamp's two-dimensional analysis can be replaced by a one-dimensional approach—if we include an instant constant n in the UT calculus. Second, Prior argues that if we move into hybrid logic and turn this instant constant into a *propositional* constant n, it is easy to capture the logic of J in a way that reflects Castañeda's central concerns. As we shall see, Prior was right. His hybrid logic for 'now' is built squarely upon Castañeda's work. Moreover, his hybrid logic of 'now' *is* complete, as we shall show using a simple semantic argument that draws on contemporary results.

2 Setting the Stage

Kamp's hand-written pragmatics notes from the fall of 1967 are only eight pages long. A crucial idea is that "['now'] always refers to <u>the moment of utterance of the sentence in which it occurs</u>" (1967, p. 2). Let J be a 'now'-operator and G the ordinary Priorean box-operator for always-in-the-future. It's a natural consequence of

[2] Prior's paper was originally published as "'Now'" in *Noûs*, 2:101–119, 1968. An addendum, correcting a technical glitch, appeared as "'Now', corrected and condensed", in *Noûs*, 2:411–412, 1968. A combined version (in which Prior's Polish notation was converted to standard notation) appeared under the title "'Now'" in the new edition of *Papers on Time and Tense* (Prior 1968/2003). In this paper, all quotations from and page references to "'Now'" are to this combined version.

Kamp's idea that $\varphi \supset GJ\varphi$ should always be true *when uttered*: if you're reading this sentence, then it's always going to be the case that you're right now—*at this very moment*—reading this sentence.[3] Moreover, $\varphi \equiv J\varphi$ should also be a validity of this kind. Given this, Kamp observes that it's *not* possible to handle J in a completely standard way semantically. In particular, we *can't* extend to J the ordinary recursive definition of modal satisfiability $\models$ in models $\mathfrak{M} = (T, \prec, V)$, where T is the set of times, $\prec$ the temporal precedence relation, and V a valuation function. Why not?

The problem, as Kamp puts it, is this: "what we need is not just a definition of 'φ is true at i' but the more complicated notion 'φ is true at i when occurring in a sentence uttered at j'" (Kamp 1967, p. 4).[4] Kamp overcomes this problem by extending the satisfiability definition with a second index: "[O]ur points of reference should be pairs" (1967, p. 4), and so we arrive at:

$$\mathfrak{M}, t, t_0 \models \varphi.$$

The idea is that the second index records the moment of utterance, whereas the first index does the work usually done by the point of evaluation in a one-dimensional semantics. Thus, for any $t \in T$, we have $\mathfrak{M}, t, t_0 \models J\varphi$ iff $\mathfrak{M}, t_0, t_0 \models \varphi$. And, as we have just mentioned, the first index takes care of the ordinary Priorean tenses in the familiar fashion. So $\mathfrak{M}, t, t_0 \models G\varphi$ iff for all t' such that $t \prec t'$ we have that $\mathfrak{M}, t', t_0 \models \varphi$. From this it follows that $\varphi \equiv J\varphi$ is true whenever it's uttered, as we always have $\mathfrak{M}, t_0, t_0 \models \varphi \equiv J\varphi$.

The upshot of all this is that it gives rise to an interesting logic. *Indeed, it gives rise to two*. First there is the standard logic, which consists of all formulas that are true at all points in all models (in this paper we call such formulas *logically valid*). But there are also expressions which are true whenever they are uttered—even though they *aren't* true at all points in all models (in this paper we shall call such formulas *contextually valid*). To give a simple example: if φ is false at (t, t_0) but true at (t_0, t_0) it follows that $J\varphi \supset \varphi$ is false at (t, t_0): suppose you won't be reading this sentence *tomorrow*, then tomorrow it will be false that, *if* you're reading this sentence right now (which you actually are) *then* you're reading this sentence (tomorrow, which you aren't). Whatever you do right now, you don't have to do forever. So $J\varphi \supset \varphi$ is contextually valid but not logically valid.

Hence Kamp's logic is 'two-dimensional' in two importantly different senses. First there is the obvious sense: his semantics makes use of evaluation at a pair of points. As we shall see, Prior showed that by changing the semantic setup in a different way, it is possible to work with a one-dimensional semantics, where one-dimensional means that we evaluate formulas at a single point. But there is a deeper

[3] Prior's own example is: "If I am sitting down, then it will always be the case that I am now sitting down" (Prior 1968/2003, p. 173).

[4] Even more specifically, the problem can be viewed the following way as Kamp (1971, p. 238) later did. As φ and $J\varphi$ should be semantically equivalent *in the sense just mentioned*, we arrive at the following problem. Suppose we're are looking for $>$something$<$ such that $\mathfrak{M}, t \models J\varphi$ iff $>$something$<$, then this $>$something$<$ would have to be equivalent to the condition for $\mathfrak{M}, t \models \varphi$ (as φ and $J\varphi$ should be equivalent whenever φ is uttered). Bearing this in mind, consider any model with at least two points, t_1 and t_2, such that $t_1 \prec t_2$, and with p true at t_1 but false at t_2. Such a model falsifies $p \supset GJp$ at t_1.

sense of two-dimensionality in Kamp's work, and this is where the beauty of his analysis lies: we have not one logic but two. Logical pluralism? It's here already, echoing indexicality in natural language! And this deeper sense of two-dimensionality, which emerged in the heady atmosphere of Richard Montague's UCLA pragmatics seminar, is the logical gem that the best subsequent work has polished. Kamp finishes his notes by posing some open questions, the most important one being an axiomatisation of the 'now'-logics.

3 Prior's Response

It is easy to see why Kamp's notes caught Prior's attention. They mark a major advance, both technically and conceptually. As he puts it in his *Noûs* paper:

> [U]ntil recently I would have [. . .] said that the formalist not only *can* do without the idiomatic 'now' but *must* do without – that our ordinary use of now has a fundamental disorderliness about it which makes it unamenable to formalisation [. . .] Recently, however, I have been convinced to the contrary by Hans Kamp (Prior 1968/2003, p. 174)

Moreover, Prior remarks that he formerly thought that adding 'now' to tense logic would have a "quite explosive effect" (1968/2003, p. 176). Why explosive? Because Kamp's work seems to call into question Prior's redundancy approach to the present.

Tense logic is fundamentally about the present. It uses tense operators to relate past and future information to present information—and every *when* is (so to speak) a present, endlessly reflecting other presents via the tense operators. Now, if you have such a vision of modal semantics, and something like this underlies both Prior's work and (in a slightly different guise) Kripke's model theoretic account, it must have been both exciting and disturbing to learn that 'now' cannot safely be ignored. Kamp unequivocally showed that 'now' was much more than a disorderly near-synonym for the present. It gives rise to genuinely new logic, and this logic cannot be straightforwardly accommodated within Prior's tense logic—it demands novel semantic ideas, such as two-dimensional evaluation. The arrival of Kamp's notes seems to have affected Prior like the ringing of an alarm clock, and it led to some of his best work.

The first and third sections of "'Now'" discuss the problems Kamp raised. The first, entitled *The redundant present and the non-redundant 'now'* (pp. 171–4), is devoted to issues of English usage. And the hard work of December 1967 has clearly paid off, for by the time he came to write the paper, Prior was able to respond forthrightly to the linguistic challenge:

> There is surely no need for prolonged agonising about all this. [. . . A]s far as English idiom goes, it seems clear that constructions involving the word 'present' fit a redundancy theory fairly well, that ones involving the word 'now' do not fit it at all well, and that ones involving the plain present tense or the plain 'it is the case that' are in between. (Prior 1968/2003, p. 173)

The third section, entitled *The resistance of tense logic to the idiomatic 'now'* (pp. 176–80), discusses logical difficulties of the kind we have already noted. Prior was acutely aware of just how logically explosive J could be when added to tense logic. For example, on p. 179 he discusses one possible repair strategy, namely weakening the equivalences $Jp \equiv GJp$ and $Jp \equiv HJp$ to $Jp \supset GJp$ and $Jp \supset HJp$, respectively. As he concluded, this is unpalatable:

> This still makes tense operators *almost* vacuous, and it seems to have this effect even with a minimum of assumptions about the character of the earlier-later relation, so a tense logician could very well be pardoned for refusing to admit such an operator as J into his system. (Prior 1968/2003, p. 180)

Prior notes that the trouble can be pinpointed to two standard tense logical rules of proof, RG and RH, the generalization rules for G and H respectively:

$$\text{RH: } \vdash \alpha \text{ implies} \vdash H\alpha \qquad \text{RG: } \vdash \alpha \text{ implies} \vdash G\alpha.$$

The minimal tense logic K_t could accommodate J if we banned these rules from applying to formulas containing J. But: "This is still, however, a tall order if we are to think of K_t as being embeddable in an an earlier-later calculus in the usual way" (p. 180).[5] As we saw earlier, Kamp observed that it's *not* possible to handle J in a semantically standard way, which is what led Kamp to change the usual recursive definition of modal satisfiability in a model to a two-dimensional version. Prior is here saying essentially the same thing: embeddability in the UT calculus (his earlier-later calculus) was Prior's analog of Kripke semantics—and this quotation is his acknowledgement that to cope with J, *something* had to be changed. The question is: *what*?

So in the fourth section, entitled *A revised earlier-later calculus to accommodate 'now'* (pp. 180–3) Prior turns to the UT calculus. Expressions of the form $Ta(\varphi)$ are a key construct in the UT calculus: they state that the formula φ holds at the instant named a (that is, φ is true at a). Prior introduces a new form $Tab(\varphi)$, where a performs the same task as the first index in Kamp's semantics, and b performs the same task as Kamp's second index (recording the moment of utterance). So we can gloss $Tab(\varphi)$ as φ is true at the pair (a, b). Prior has simply recreated Kamp's two-dimensional approach in his UT calculus.

And Prior is dissatisfied with this. He wants to integrate J into a one-dimensional tense logic—and his previous work on hybrid logic suggests a way through. However, the hybrid approach requires an element of *temporal reference*, and it's here that Castañeda comes into the picture. The alert reader will have noted that we have not yet said anything about the second section of the paper, which is entitled *The elimination of the idiomatic 'now'* (pp. 171–4). In this section Prior discusses Castañeda's work. Castañeda seems to have legitimatized at least some forms of temporal reference in Prior's eyes—and temporal reference was a concept that Prior (with his well-known dislike of instants) was never entirely comfortable with.

So Prior brings in Castañeda (1967). In oblique contexts 'now' has—as an analogue to 'I'—a special *referential* function.[6] Castañeda provides a thorough philosophical analysis of words like 'I', 'now', and 'here' which function as what he calls

[5] To put Prior's point in contemporary terms—it's all very well saying that we should just ban such rule applications, but can this be done without sacrificing completeness?

[6] Prior mentions a number of examples originating from (Castañeda 1967), two of them being "It will be the case *tomorrow* that it is now the case that I am sitting down" (Prior 1968/2003, p. 172) and "I think Brown thinks I can help him" (Prior 1968/2003, p. 175).

"indicators" and "quasi-indicators". When discussing the general function of these words, Castañeda says that a "certain mechanism of reference to particulars [. . .] is a unique logical category of reference" (Castañeda 1967, p. 86). Thus, *reference* plays a central role in the Castañeda analysis. Moreover, he continues that: "Reference to an entity by means of an indicator is purely referential, i.e., it is a reference that attributes no property to the entity in question" (Castañeda 1967, p. 85).

So (with the backing of Castañeda) Prior is philosophically prepared to follow the path that will lead him (via hybrid logic) to a one-dimensional semantics for J. The first step involves a simple modification of the UT calculus:

> Kamp's way is not the only way of modifying the UT calculus to obtain one that will accommodate J; though it is, so far as I know, the first solution offered to that problem. We might instead keep the simple form $Ta(p)$ [. . .] but introduce a constant, say n, for a particular instant (Prior 1968/2003, p. 182)

Prior has no qualms about talking about instants of time, and referring to them with constants, *in the UT calculus*—indeed, the whole point of the UT calculus is to provide a B-series perspective on time. And with two-dimensionality out of the way, Prior sees a way that will lead all the way through to a one-dimensional *tense* logic for J: introduce n as a special *propositional* constant into a tense logic (that is, into an A-series logic). Prior realizes he can adapt a system from C.A. Meredith's work on modal property calculus, which makes use of "a contingent constant n for 'the world' in the sense of 'everything that is the case'. And we do know how to axiomatise that" (p. 183).[7] So, accompanied by Castañeda and Meredith, Prior begins the brief intermezzo (it takes ups less than a page) which will now occupy our attention: he switches from the UT calculus to hybrid logic. He introduces n as a special *propositional* constant, states a number of axioms, and remarks:

> The postulates RL (modified), L1–5 and A1.1–A5 with the definition of J, will I think yield all the theorems in J that we want; and the definition of 'now' as expressing contemporaneity with some unspecified proposition which is true only at the time of utterance nicely formalises Castañeda's explanation of the use of 'now' in oblique contexts. (Prior 1968/2003, p. 184)

Then, with the following (almost apologetic) words, he moves on from hybrid logic to a non-referential system (tense logic enriched with $\Box$ and J):

> It may be felt, however, that this system is too much of a hybrid between a UT calculus and a tense logic. (Prior 1968/2003, p. 184)

Memorable words—and probably the first time the word 'hybrid' has been used in connection with such logics![8] But they signal the end of the hybrid intermezzo. With

[7] Prior cites their joint paper (Meredith and Prior 1965) which deals with axiomatisation, but notes that Meredith's work on the subject dates back until at least 1953.

[8] Prior did not speak of hybrid logic; that term only gained currency in the 1990s, long after Prior's death; the term was first used in passing in (Blackburn 1990), and the publication of Blackburn and Seligman (1995) was the baptismal event. Prior regarded hybrid logic as a part of tense logic, indeed it was the *third grade of tense logical involvement*, as he explained in his paper "Tense Logic and the Logic of Earlier

the hybrid axiomatisation in place, hybrid logic's role as a bridge between the UT calculus enriched with a constant n and the (ordinary, non-referential) one-dimensional logic in $\Box$ and J is over, and it can be put aside again.

But Prior's remark ("will I think yield all the theorems in J that we want") is essentially a modestly stated completeness conjecture. And, as we shall show, it *is* correct. So we will now dive back into the pool that Prior has so hastily vacated and show how prescient he was.

4 A Hybrid Tense Logic for 'Now'

Hybrid tense logic is a simple extension of ordinary Priorean tense logic in which it is possible to refer to times. It does this using special propositional symbols which nowadays are called *nominals*. Nominals are true at one and only one time: they 'name' the time they are true at.[9]

We will work in the same hybrid logic that Prior uses in "'Now'", that is, a language built on a set Nom of nominals (typically written i, j and k) and a set Prop of ordinary propositional symbols (typically written p, q and r). As connectives we take some truth functionally adequate collection of boolean operators (here we'll follow Prior and choose $\sim$ and $\supset$) and the symbols H, G, and $\Box$. Formulas are built as follows:

$$\varphi ::= i \mid p \mid {\sim}\varphi \mid \varphi \supset \psi \mid H\varphi \mid G\varphi \mid \Box\varphi.$$

We define $P\varphi$ to be ${\sim}H{\sim}\varphi$, $F\varphi$ to be ${\sim}G{\sim}\varphi$ and $\Diamond\varphi$ to be ${\sim}\Box{\sim}\varphi$

Now, as we have mentioned, Prior's main formal semantic tool was the UT calculus. But the favoured contemporary semantics (and the semantics Kamp used) is Kripke semantics, so let's continue model theoretically. As we have already mentioned, models $\mathfrak{M}$ are triples $(T, \prec, V)$, with T a set of times and $\prec$ the earlier-later relation. However, we won't impose any conditions (such as transitivity, or irreflexivity, or linearity) to make the $\prec$ relation more time-like; we are going to follow Prior and work with the minimal logic.[10]

The valuation function V, takes all atomic symbols (that is, both nominals and ordinary propositional symbols) to subsets of points of T. Ordinary propositional symbols are unrestricted in their interpretation: they encode arbitrary information, such as when it was sunny in Masterton, or the timing of the All Blacks' test victories.

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and Later", which also can be found in (Prior 1968/2003). As the phrase "too much of a hybrid " seems to suggest, Prior's views on hybrid logic (third grade tense logic) were somewhat equivocal; see (Blackburn 2006) for further discussion.

[9] That Prior was the inventor of hybrid logic is surprisingly little known given the central role they play in his work on temporal logic; this curious state of affairs is discussed in detail in (Blackburn 2006). In the present paper we have (by and large) adopted contemporary hybrid logical notation and terminology; for example, Prior would have spoken of world-variables rather than nominals. But to make the comparison with "'Now'" more transparent we have followed Prior and used $\Diamond$ and $\Box$ for the universal modalities.

[10] Prior is insistent here: "For the present, however, let us simply consider the system K_t which is in a sense 'minimal'. It is well to confine ourselves to this because I want to show that it is awkward to introduce into tense-logic an operator with the properties of the idiomatic 'now', but if the tense-logic into which I introduce this operator is richer than K_t it is too easy to suggest that the trouble arises from my having made rash assumptions about time in the first place." (Prior 1968/2003, p. 178).

But we place a crucial restriction on the valuation $V(i)$ of any nominal i: this must be a *singleton* subset of T. So nominals in effect are names for time in T.

Given a model $\mathfrak{M} = (T, \prec, V)$ we define truth at a time as follows:

$$
\begin{array}{lll}
\mathfrak{M}, t \models a & \text{iff} & a \text{ is atomic and } t \in V(a) \\
\mathfrak{M}, t \models \sim\varphi & \text{iff} & \mathfrak{M}, t \not\models \varphi \\
\mathfrak{M}, t \models \varphi \supset \psi & \text{iff} & \mathfrak{M}, t \models \varphi \text{ implies } \mathfrak{M}, t \models \psi \\
\mathfrak{M}, t \models H\varphi & \text{iff} & \text{for all } t',\ t' \prec t \text{ and } \mathfrak{M}, t' \models \varphi \\
\mathfrak{M}, t \models G\varphi & \text{iff} & \text{for all } t',\ t \prec t' \text{ and } \mathfrak{M}, t' \models \varphi \\
\mathfrak{M}, t \models \Box\varphi & \text{iff} & \text{for all } t',\ \text{we have } \mathfrak{M}, t' \models \varphi.
\end{array}
$$

A word on the role played by $\Diamond$ and $\Box$. Note that $\Box\varphi$ claims that φ is true at all times, while its dual form $\Diamond\varphi$ scans the model looking for a time where φ is true. Consider the following two schemas:

$$\Box(i \supset \varphi) \qquad \Diamond(i \wedge \varphi).$$

These are equivalent. The first says: at every time where i is true, φ is true too. The second says: there is a point where i is true and φ is true there too. In short, these are modal expressions which allow Prior to capture the effect of UT calculus formulas of the form $Ti(\varphi)$.[11]

But where is 'now'? Easy! Just add a brand new nominal n to the language (or re-christen one of the old ones if you prefer). Then, given any model $\mathfrak{M} = (T, \prec, V)$ pick some $t_0 \in T$ and regard this designated time as the 'now' of the model. Insist that in any model, n must be true at t_0 (and nowhere else). It's precisely this idea Prior had in mind, when he on the 11th of December 1967 wrote to Kamp that "a model for my J is not difficult to obtain by having a 'designated instant' (as, e.g. Kripke has in his modal models [...])" (Prior 1967, p. 2) . Given this, it's immediately clear that $\Box(n \supset \varphi)$ and $\Diamond(n \wedge \varphi)$ both state that φ is true now, and these are the formulas that Prior uses to *define* his 'now' operator J.

So $J\varphi$ works like a Kamp-style 'now' operator, but it is defined in a one-dimensional semantics. But of course, it is not a completely standard Kripke semantics. Just as Kamp had to introduce two-dimensional evaluation to provide sufficient semantic space for 'now', Prior made space by introducing a special referential propositional symbol. As we've noted before: *something* has to be changed, the question is: *what*? Kamp opted for two-dimensionality. Prior opted for Castañeda and temporal reference. Concerning the choice, Prior in fact wrote Kamp on the 11th of December 1967, that "I think my J catches the real forms of 'now' better than your N" (Prior 1967).

[11] These operators are nowadays often written E (there exists some time) and A (at all times) and are usually called universal modalities. They have played an important role in the development of hybrid logic (see (Gargov and Goranko 1993) and (Blackburn and Seligman 1995)) and are important in their own right (see (Goranko and Passy 1992)). Nowadays the notation $@_i\varphi$ is standardly used to state that φ is true at the time named by the nominal i. This is sometimes introduced as a primitive symbol, and sometimes as shorthand for $\Diamond(i \wedge \varphi)$ and $\Box(i \supset \varphi)$.

5 Why Prior Was Right

Prior then distinguishes between the two types of validity that we have already noted. In the *Noûs* paper he puts it like this: "We then seek for those tense-logical formulae which can be proved attachable either to any arbitrary instant a, and so to n, or just to n." (Prior 1968/2003, p. 183). In fact this distinction goes back to C.A. Meredith who introduced it around 1953. It's found in (Meredith and Prior 1965) as "universal validity" versus "accidental validity". For our reconstruction within possible world semantics, we will say (as noted earlier) that a formula is *logically valid* iff it is true at all points in all models, and *contextually valid* iff it is true in all models at the designated points t_0. Clearly all logically valid formulas are contextually valid, but there are contextually valid formulas that are not logically valid: n and $J\varphi \supset \varphi$ are two easy examples. Moreover, we say that a proof system is *logically complete* iff it generates all logical validities and *contextually complete* iff it generates all contextually valid formulas.

It's now time to come back to Prior's remark: "RL (modified), L1–5, A1.1–A5 with the definition of J, will I think yield all the theorems in J that we we want". Prior's axiomatisation consists of 12 axioms and one rule (a complete listing is given in the Appendix). Some are more-or-less obvious, for example the familiar

A1.1. $G(p \supset q) \supset (Gp \supset Gq)$.

But as Prior is interested in axiomatising the contextual validities he also adds

A3. n

as an axiom. Thus he needs to be careful with the generalisation rule for the boxes, so the rule

RL: $\vdash \alpha$ implies $\vdash \Box\alpha$,

is accompanied by the side condition "provided that α does not contain J or n" (Prior 1968/2003, p. 184).[12] And in Prior's axiomatisation (see the Appendix) generalisation with G and H goes via RL and two axioms, L4 $\Box p \supset Gp$ and L5 $\Box p \supset Hp$, so the restriction put on RL ensures that we cannot derive the unsound Gn (at all points in the future, n will be true).

Prior's completeness conjecture is correct. To prove it we split it into two claims.

Logical Completeness. Take Prior's axiomatisation but *without* A3, which is just n, the simplest axiom of all. This system is logically complete. Why? First, these axioms and rules are closely related to complete proof systems for hybrid logic with $\Diamond$, and it easy to check their adequacy; see, for example, the systems in (Gargov and Goranko 1993). Second, it is straightforward to check that interpreting the special n nominal only on the designated point t_0 affects nothing.[13] Technical details for related systems can be found in (Blackburn and Jørgensen 2012, 2013), but the argument is

[12] Recall Prior's early comment on restricting the generalisation laws: "this is a tall order if we are to think of K_t being embeddable in an an earlier-later calculus in the usual way" (p. 180). Well, he has successfully filled his tall order, and done so by linking with the UT calculus in an *unusual* way, namely via hybrid logic.

[13] One remark. Recall the restriction put on RL. From $\vdash \varphi$ we can conclude $\vdash \Box\varphi$, given that φ does not contain occurrences of n or J. This restriction does *not* affect logical completeness. If φ is a logical validity containing occurrences of n or J, then choose a nominal k not occurring in φ and replace all

straightforward: as far as *logical* validity is concerned, n is interchangeable with any other nominal.

Contextual Completeness. So we have a logically complete system. Add to it A3, that is n, and we have contextual completeness. Why? First a simple fact which we leave the reader to check: any formula φ is contextually valid iff $\Box(n \supset \varphi)$ is logically valid. But this means that we could prove any contextual validity φ if we could first prove $\Box(n \supset \varphi)$ and then "peel away" the $\Box(n \supset \quad)$ to reveal φ. And we *can* do this: the logic of $\Box$ is **S5**, hence from $\vdash \Box(n \supset \varphi)$ we can prove $\vdash n \supset \varphi$ and then, using A3 and Modus Ponens, we have φ. This establishes contextual completeness. Checking *soundness* almost takes more effort, as here the restrictions on RL come into play.

So Prior's bridge from the B-language world of the UT calculus to the A-language world of tense logic is sturdily built. And with that crossing made, Prior bids farewell to hybrid logic, and moves on to build a (non-referential) one-dimensional tense logic for J (with the help of $\Box$).

6 Conclusion

This paper has not covered all that is of interest in "'Now'". To mention some: Prior's axiomatisation of tense logic enriched with $\Box$ and J (the very step our discussion has just reached) is of independent interest, as is Meredith's property calculus axiomatisation (given on p. 188). Moreover, the paper raises questions of relevance to contemporary work on temporal indexicals and actuality. What are the tradeoffs between two-dimensional (or multidimensional) approaches and the referential approaches? And are there significant differences between purely operator-driven and hybrid approaches to indexicality and actuality? (For some answers to this latter question, see (Blackburn and Marx 2002)). Finally, our discussion clearly indicates the need for more historical research. Meredith's property calculus was one of the stepping stones—perhaps *the* major stepping stone—on the way to Prior's hybrid logic, but we are woefully short on historical detail here.

But one of the most interesting aspects of "'Now'" is the role played by temporal reference, and Castañeda's influence. Although Prior invented hybrid logic—which nowadays is regarded as a form of modal logic in which reference to times, worlds, epistemic states, or anything of interest, is possible—Prior did *not* think of hybrid logic in this way. For Prior, hybrid logic played a largely theoretical role. He created it to show that there was an A-series language powerful enough to express everything that the UT language (his B-series language) could express. Hence Prior needed strong hybrid languages with which he could quantify over nominals—or as he would have put it, world variables. And given Prior's goals, his terminology is far more suitable than contemporary usage: for Prior, hybrid logic provided a fundamentally *quantificational* syntax, not a *referential* one.

Now, occasionally Prior explored hybrid logic's referential potential. For example, in "Egocentric Logic" (reprinted in *Papers on Time and Tense* (Prior 1968/2003))

occurrences of n by k. This new formula, $\varphi[n \leftarrow k]$, is also logically valid and hence provable. But then we prove φ in one more step by substituting n for k.

he uses it for reasoning about people. However, Prior never took such applications very far, and (with the apparently unique exception of "'Now'") never seems to have used them for temporal reference. His dislike of instants, and his world-variables-are-for-quantifying perspective kept him away from temporal reference, so he never saw, for example, the relevance of hybrid logic to Reichenbach's referential account of tense.

But Castañeda seems to have opened his eyes to new possibilities. Recall the quotation we met earlier:

> Reference to an entity by means of an indicator is purely referential, i.e., it is a reference that attributes no property to the entity in question. (Castañeda 1967, p. 86)

It is hard to think of a more apt description of the modern *referential* perspective on nominals: the modern model theoretic semantics simply assigns a single time as the denotation to a nominal—and the nominal certainly attributes no property to the entity in question.

How wholeheartedly did Prior grasp the possibilities opened by Castañeda? Were Castañedas ideas only useful for justifying referentiality for certain special constants like 'now', or did they open the door to more general forms of temporal reference? Were the lines of development hinted at in late papers such as "Egocentric Logic" and "'Now'" a harbinger of new things to come, or one-shot experiments? Hopefully further archival research will reveal more here too.

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Appendix: Prior's System for Now

Here we present Prior's axiomatisation of his tense hybrid logic of 'now'. The primitive symbols in the language are $\sim$, $\supset$, H and G. The other symbols $\vee$, $\wedge$, $\equiv$ and P, F are defined in the usual way. For propositional logic, Prior takes any complete propositional axiomatisation (so we may assume we have the rule Modus Ponens either as a primitive or derived proof rule). To this he then adds the following 12 axioms and the modified RL rule (in A3, n is the special symbol for 'now', and in A4 and A5, i is a nominal):

A1.1.	$G(p \supset q) \supset (Gp \supset Gq)$
A1.2.	$H(p \supset q) \supset (Hp \supset Hq)$
A2.1.	$PGp \supset p$
A2.2.	$FHp \supset p$
A3.	n
A4.	$\sim\Box\sim i$
A5.	$\Box(i \supset p) \vee \Box(i \supset \sim p)$
RL:	If $\vdash \alpha$ then $\vdash \Box\alpha$, provided that α does not contain n
L1.	$\Box p \supset p$
L2.	$\Box(p \supset q) \supset (\Box p \supset \Box q)$
L3.	$\sim\Box p \supset \Box\sim\Box p$
L4.	$\Box p \supset Gp$
L5.	$\Box p \supset Hp$

Included in this axiomatisation is a rule of substitution allowing one to substitute nominals for nominals and arbitrary formulas for propositional symbols. Prior defines his Kamp-style 'now'-operator $J\varphi$ to be $\Box(n \supset \varphi)$. As noted in the text, G and H generalization goes via RL: if φ is provable, then by RL so is $\Box\varphi$. Thus from axiom L4 we get $G\varphi$, and from L5 we get $H\varphi$. That is, both RG and RH are derived rules.