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Reichenbach, Prior and Hybrid Tense Logic

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Abstract In this paper we argue that Prior and Reichenbach are best viewed as allies, not antagonists. We do so by combining the central insights of Prior and Reichenbach in the framework of hybrid tense logic. This overcomes a well-known defect of Reichenbach's tense schema, namely that it gives multiple representations to sentences in the future perfect and the future-in-the-past. It also makes it easy to define an iterative schema for tense that allows for multiple points of reference, a possibility noted by Prior and demanded by Comrie, and we sketch how this schema can be generalized to a shift-and-restrict pattern in which special propositional symbols (for adverbials and indexicals) act as restrictors on the range of tense operators.

Keywords Arthur Prior · Hans Reichenbach · point of reference · hybrid logic

1 Introduction

Arthur Prior's *Past, Present and Future* (Prior 1967), henceforth *PPF*, contains the first detailed work on hybrid logic, an extension of ordinary tense logic which allows reference to times using special propositional symbols. That Prior was the inventor of tense logic is well known; that he was also the inventor of hybrid logic is not. Nonetheless, hybrid logic plays a clear (if somewhat restricted) role in Prior's best

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known book.[1] But it could have played a bigger one, for *PPF* is also the source for Prior's views on Reichenbach.

Prior's discussion of Reichenbach's referential theory of tense can be found in Chapter 1, Section 6, of *PPF*, "Reichenbach on the time of speech and the time of reference; the nature of presentness". After a brief sketch of Reichenbach's approach, Prior announces that "Reichenbach's scheme, however, will not do as it stands; it is at once too simple and too complicated" (Prior 1967, p. 13). Why too simple? Because in Reichenbach's scheme, every tense has just one reference point, and Prior devises an example which requires two. Why too complicated? Because Prior immediately claims that Reichenbach has missed a fundamental generalisation. We shall discuss Prior's claim later in the paper—it's interesting and important—but then Prior appears to switch topic: for the remainder of the section he reverts to discussing Findlay's pioneering work on tense formation (Prior 1967, pp. 14–15).[2] Reichenbach is not mentioned in this part of the discussion (which does indeed concern "the nature of presentness") but we read it as an amplification of Prior's "at once too simple and too complicated" critique. First, Prior claims that Reichenbach has missed an essential logical insight about tense (which Prior attributes to Findlay) namely that tense (like negation) is an *iterable* propositional constructor, one that transforms formulas to formulas; as is well known, this idea is central to Priorean tense logic. But then Prior goes further: he also seems to claim that the intermediate times encountered in the course of evaluating formulas containing iterated tense operators embody the key idea of Reichenbach's reference points.

We return to this point later in the paper. However, our main purpose here is not exegetical, rather it is to combine the Reichenbachian and Priorean approaches to tense. As we shall see, Reichenbach's approach has some well-known shortcomings. But Prior also has a blind-spot: he underestimates the importance of *explicit* temporal reference in natural language. This is unfortunate, for the hybrid logic he presented in *PPF* gave him the tool he needed not merely to make explicit the (fairly limited) types of temporal reference used in Reichenbach's analysis of tense, but to make temporal reference both explicit and iterable. Indeed, using Prior's own hybrid logic to combine the operator-based approach to tense with the temporal reference approach (pioneered by Reichenbach) leads to possibilities not considered by either author; we will give examples involving dates, times, indexicals, and temporal anaphora.

Our use of hybrid logic has much in common with ideas developed by the linguist Bernard Comrie, first in his paper "On Reichenbach's Approach to Tense" (Comrie

[1] The material most directly related to modern hybrid logic can be found in Chapter 5, Section 6, "Development of the U-calculus within the theory of world-states" (Prior 1967, pp. 88–92) and Appendix B, Section 3, "On the range of world-variables, and the interpretation of U-calculi in world-calculi" (Prior 1967, pp. 187–97). The discussion is a largely technical explication of the relationship between hybrid-style tense logics and the U-calculus (Prior's B-series language of time). However, other material in Chapter 5 is also relevant: for example, in Section 2, "Instantaneous world states", he notes a range of closely related ideas such as Meredith's constant for the present. Prior's more detailed expositions and explorations of hybrid logic can be be found in several papers in the 1968 edition of *Papers on Time and Tense* (Prior 1968/2003) including "Tense logic and the logic of earlier and later" and "Quasi-propositions and quasi-individuals"; the 2003 edition also contains " 'Now' " and "Egocentric logic".

[2] Prior introduces Findlay's work in Chapter 1, Section 5, of *PPF*, "Findlay's tense-logical laws", the section immediately preceding his discussion of Reichenbach.

1981) and later in his textbook *Tense* (Comrie 1985). Comrie's paper is a detailed and fundamentally sympathetic critique of Reichenbach's work, packed with linguistic detail. His textbook incorporates the insights of the paper, and concludes with a Reichenbach-inspired schema intended to depict the general (iterative) nature of tense in natural language. Towards the end of our paper we will give a Comrie-inspired schema in the language of hybrid logic.

Summing up, this paper, which builds on Blackburn (1990, 1994), brings together Reichenbach and Prior's central insights in a simple logical framework—a framework invented by Prior himself. It makes Reichenbach's referential insights part of a system in which iteration is inbuilt, and adds explicit temporal reference to the Priorean approach. Prior and Reichenbach are best viewed as allies, not antagonists.

2 Hybrid Tense Logic

Hybrid tense logic is a simple extension of ordinary Priorean tense logic which can refer to times. It can do so because it contains a collection of special propositional symbols called *nominals*. Nominals are true at one and only one time: they 'name' the time they are true at. We will now present hybrid logic using contemporary notation and terminology, and give a Kripke-style truth definition in terms of models. After our presentation we briefly contrast our approach to hybrid logic with Prior's.

A language $\mathcal{L}$ of basic hybrid tense logic is built over denumerable sets Nom (of nominals) and Prop (of ordinary propositional symbols); we usually indicate nominals by i, j and k and ordinary propositional symbols by p, q and r. Nominals and propositional symbols are the *atoms* of the language. As connectives we take the boolean operators $\neg$ and $\wedge$, the two existential Priorean tense operators F and P, and an $@_i$-operator for each nominal i. Formulas of $\mathcal{L}$ are built as follows:

$$\varphi ::= i \mid p \mid \neg\varphi \mid \varphi \wedge \psi \mid P\varphi \mid F\varphi \mid @_i\varphi.$$

Note that any nominal i can occur syntactically in two distinct ways: in *formula position* as the atomic symbol i, or in *operator position* as in $@_i\varphi$. Such an $@$-operator is sometimes called a *satisfaction* operator.

Models $\mathfrak{M}$ are based on frames $(T, \prec)$; here T is a set of instants of time and $\prec$ is the temporal precedence relation. When reasoning about temporal problems we typically demand that $\prec$ be irreflexive and transitive, and sometimes we impose even more structure (such as linearity, density or discreteness). But here we will view $\prec$ as an *arbitrary* binary relation. That is, we will be working in the *minimal* hybrid tense logic. We do this in order to show that our account of Reichenbach's use of reference points does not require us to make additional assumptions about temporal order.

To fully specify a model we also need an *information distribution*, together with a *specification of names* for times of interest. Both tasks are performed by a valuation function V, which takes ordinary propositional symbols and nominals to subsets of T. Ordinary propositional symbols are unrestricted in their interpretation: they encode arbitrary information. But nominals bear name-like information hence we insist that the valuation $V(i)$ of any nominal i is a *singleton* subset of T. Given a nominal i, we sometimes call the unique $t \in V(i)$ the *denotation* of i.

For all formulas of hybrid tense logic, all models $\mathfrak{M} = (T, \prec, V)$ and all $t \in T$ we define truth as follows:

$$
\begin{array}{lll}
\mathfrak{M}, t \models a & \text{iff} & a \text{ is atomic and } t \in V(a) \\
\mathfrak{M}, t \models \neg\varphi & \text{iff} & \mathfrak{M}, t \not\models \varphi \\
\mathfrak{M}, t \models \varphi \wedge \psi & \text{iff} & \mathfrak{M}, t \models \varphi \text{ and } \mathfrak{M}, t \models \psi \\
\mathfrak{M}, t \models P\varphi & \text{iff} & \text{for some } t',\ t' \prec t \text{ and } \mathfrak{M}, t' \models \varphi \\
\mathfrak{M}, t \models F\varphi & \text{iff} & \text{for some } t',\ t \prec t' \text{ and } \mathfrak{M}, t' \models \varphi \\
\mathfrak{M}, t \models @_i\varphi & \text{iff} & \mathfrak{M}, t' \models \varphi \text{ where } t' \text{ is the unique time in } V(i).
\end{array}
$$

Most of this is familiar from ordinary tense logic. In particular, $F\varphi$ scans the future looking for a time where φ is true, and $P\varphi$ scans the past. What is new is the role played by nominals and satisfaction operators. First, as an atom a can be either a nominal or a propositional symbol, the first clause of the definition handles both sorts of symbol, hence the fundamental restriction on the interpretation of nominals is built into the core of the truth definition. Next, note that $@_i\varphi$ is true at a time in a model $\mathfrak{M}$ if and only if φ is true at the denotation of i in $\mathfrak{M}$. So to speak, $@_i\varphi$ peeks at the time named i (and there *must* be such a time because of the restriction imposed on the interpretation of nominals) and checks whether φ is true then or not.

Hybrid tense logic is simple and well-behaved. We remarked above that in a temporal setting it is natural to demand that $\prec$ be irreflexive and transitive, and that we might well want to impose additional conditions on the temporal flow. But as authors such as Bull (1970), Gargov and Goranko (1993), and Blackburn and Tzakova (1999) have pointed out, it is straightforward to axiomatize and prove completeness theorems for a wide range of such extensions—indeed, more straightforward than in ordinary tense logic. Furthermore, extending ordinary tense logic to hybrid tense logic typically does not raise the computational complexity. Over many flows of time (for example, linear time flows) the ordinary tensed language is itself capable of expressing the basic hybrid apparatus of nominals and @-operators, and even when it is not, the complexity of the hybrid logic obtained by adding them as primitives is usually identical with that of the underlying tense logic; for a detailed discussion, see (Areces et al. 2000).

No doubt this is all very well and good—but the reader may be wondering how truly *Priorean* this presentation of hybrid logic is. After all, Prior disliked instants of time—indeed, he introduced nominals to avoid having to deal with them. Nominals (or as he called them, world-state propositions or world-variables) were intended to let talk of instants be *replaced* by the use of special world- and time-identifying propositions. Consider his remark from *PPF*:

> A world-state proposition in the tense logical sense is simply an *index of an instant*; indeed, I would like to say that it *is* an instant, in the only sense in which 'instants' are not highly fictitious entities. (Prior 1967, pp. 188–189).

Surely the Kripke-style presentation given above, with its models of temporal instants ordered by precedence, and the talk of the *denotation* of a nominal, flies in the face of Prior's main motivation!

At first glance this seems like a legitimate complaint—but on closer inspection, matters are more equivocal. As Prior himself discovered, in such papers as "Quasi-Propositions and Quasi-Individuals" and "Egocentric Logic" (both in *Papers on Time and Tense* (Prior 1968/2003)) his hybrid logics were not restricted to temporal reasoning. In these papers Prior uses what he calls person-propositions: roughly speaking, he uses nominals as person-pronouns and uses the tense operators to reason about relations holding between people (such as "being taller than"). Indeed, in these papers Prior anticipates some key ideas of modern description logic, a widely used formalism for knowledge representation.[3] Thus while Prior might find talk of a nominal denoting an instant of time misguided, it is an option that his own work made possible, and indeed, made possible for a wide range of entities, not merely temporal ones. Moreover in natural language, explicit reference to times is ubiquitous: dates and times are perhaps the most obvious example, temporal indexicals are another, and (as we shall soon see) reference *via the tense system itself* to what Reichenbach called reference times is important too. But with the exception of the late paper " 'Now' " (also reprinted in (Prior 1968/2003)) Prior seems to have paid little attention to the referential aspects of natural language.[4] Viewed from the perspective of natural language semantics, this is unfortunate; doubly so, in that his hybrid logics are an excellent tool for exploring the semantics of temporal reference. Nonetheless, the reader who shares Prior's scruples about instants can dispense with the above Kripke-style presentation (or view it as a heuristic crutch) and read what follows in a more Priorean spirit: *much temporal reference in everyday discourse can be paraphrased into a purely propositional language using just iterable tense operators, special proposition symbols and satisfaction operators*.

3 Reichenbach in Hybrid Tense Logic

A little history. Prior's operator-based approach to tense in *Past, Present and Future* has many merits, chief of which is the way it builds the deictic center (or speech time) into the heart of the semantics (it's the point in the model at which we start the evaluation) and locates all events relative to this. But although tense operators were used in Montague's (1973) work, later approaches to formal semantics, notably Hans Kamp's Discourse Representation Theory (DRT), see (Kamp 1981; Kamp and Rohrer 1983; Kamp and Reyle 1993), emphasized the importance of *temporal reference*, the ability to name times and to assert relations, such as identity or precedence, between them. Reichenbach's (1947) referential analysis of tense was the inspiration here, and approaches to tense influenced by his work have largely displaced Prior's

[3] See (Blackburn 2006; Blackburn and Tzakova 1998) for more on this topic, and (Baader et al. 2003) for a detailed overview of description logic. For a general account of the relationship between description logic and hybrid logic, see (Areces 2000).

[4] Prior's important late paper " 'Now' " seems to signal a shift in Prior's attitude to temporal reference. Challenged by Hans Kamp's work on the temporal indexical *now*, and taking his philosophical cue from Castañeda's work on indicators and quasi-indicators, Prior drew on Meredith's idea of a propositional constant (a nominal) for the present to show (among other things) how Kamp's non-referential two-dimensional analysis could be replaced by a *referential* one-dimensional approach. See Blackburn and Jørgensen (n.d.) for details.

approach in natural language semantics.[5] The following quotation, from a widely cited monograph on tense and aspect, is fairly typical of the view from linguistics:

> Tense logic represents tenses by means of temporal operators. [...] we will illustrate that to correctly account for the temporal meaning of a sentence, reference to temporal entities and not just temporal operators, is required. (Giorgi and Pianesi 1997, pp. 17)

Diagram	Tense	Examples
E–R–S	Past perfect (pluperfect)	Hans had run
E,R–S	Simple past	Hans ran
R–E–S	**Future-in-the-past**	
R–S,E	**Future-in-the-past**	**Hans would run**
R–S–E	**Future-in-the-past**	
E–S,R	Perfect	Hans has run
S,R,E	Present	Hans runs
S,R–E	Prospective	Hans is going to run
S–E–R	**Future perfect**	
S,E–R	**Future perfect**	**Hans will have run**
E–S–R	**Future perfect**	
S–R,E	Future	Hans will run
S–R–E	Future-in-the-future	(Latin: abiturus ero)

Fig. 1 Reichenbach's system

So let us consider Reichenbach's system. Figure 1 tabulates possible tenses, with examples, and gives their representations using Reichenbach style diagrams.[6] At the heart of Reichenbach's system lies the idea that the utterance of a tensed sentence makes reference to *three* (not necessarily distinct) times: speech time (when the speaker speaks), event time (when what is spoken of takes place) and Reichenbach's innovation, *reference time*. To see the need for reference times, consider the sentence *Hans had run*, which is in the past perfect. When we utter this sentence, we refer to some (contextually salient) past time and locate the running *before that*. This is indicated by Reichenbach's E–R–S diagram. Similarly, consider the sentence *Hans ran*, which is in the simple past. According to Reichenbach, the function of the simple past is to locate an event in the past at the point of reference; he indicates this using the notation E,R–S. As these examples suggest, hyphens depict temporal precedence, and commas temporal coincidence.

Although influential, Reichenbach's system has been criticized by linguists on a number of grounds. Here we examine what Comrie (1981, p. 26) calls a "major defect" and show that it disappears when hybrid logic is used. Here's the defect. The above table spells out all thirteen configurations of R, E, and S that can be constructed

[5] Reichenbach's longstanding and widespread influence is even more remarkable when it is recalled that his work on tense was not presented in a paper for specialists, but as an 11 page extended example of token-reflexive symbols in a 444 page introduction to symbolic logic.

[6] This table, modulo the examples chosen, is essentially the table given on page 25 of Comrie (1981). The Latin example *abiturus ero* used to illustrate the future-in-the-future is Comrie's.

using hyphens and commas. But linguists object to the representation given to sentences in the future perfect (shown in bold; analogous remarks apply to the future-in-the-past). A sentence in this tense has only a single temporal meaning, namely that S precedes R, and that E precedes R; *the precedence relationship between S and E is unspecified*. As Comrie puts it:

> If someone asked me: Will John have finished his paper by tomorrow? and I answer: Yes, then my reply will be judged truthful (i) if John finishes the paper between the time of my reply and tomorrow (S–E–R), (ii) if John in fact finishes the paper at the moment I reply (S,E–R), or (iii) if John has in fact already finished the paper at the moment I reply (E–S–R). Of course, if I know that John has already finished his paper, it would be devious of me to assent to the question with the future perfect, rather than saying that he has already finished, but all this indicates is that the future perfect carries a conversational implicature, à la Grice, that the speaker is not certain that E has already occurred; S–E is not part of the meaning of the future perfect. (Comrie 1981, p. 26)

Comrie goes on to say that "Reichenbach's account of the Future perfect effectively claims that this form is three-way ambiguous, rather than vague", and at the end of the paper he outlines a system for capturing what is good in Reichenbach and omitting what is not. But there is an alternative: use hybrid logic.

Diagram	**Tense**	**Examples**	**Hybrid Logic**
E–R–S	Pluperfect	Hans had run	$P(i \wedge P\varphi)$
E,R–S	Past	Hans ran	$P(i \wedge \varphi)$
R–E–S	**Future-in-the-past**		
R–S,E	**Future-in-the-past**	**Hans would run**	$\boldsymbol{P(i \wedge F\varphi)}$
R–S–E	**Future-in-the-past**		
E–S,R	Perfect	Hans has run	$P\varphi$
S,R,E	Present	Hans runs	φ
S,R–E	Prospective	Hans is going to run	$F\varphi$
S–E–R	**Future perfect**		
S,E–R	**Future perfect**	**Hans will have run**	$\boldsymbol{F(i \wedge P\varphi)}$
E–S–R	**Future perfect**		
S–R,E	Future	Hans will run	$F(i \wedge \varphi)$
S–R–E	Future-in-the-future	(Latin: abiturus ero)	$F(i \wedge F\varphi)$

Fig. 2 Prior meets Reichenbach

Consider Figure 2, which adds hybrid logical representations to the previous table.[7] First consider *Hans had run*. As we said earlier, when we utter this sentence, we refer to some (contextually salient) past time, the reference time, and locate the running *before that*. But the hybrid formula $P(i \wedge P\varphi)$ does this: it asserts that there is a reference time (picked out by i) before the utterance time, and that *before that* the running took place. Similarly, the formula $P(i \wedge \varphi)$ used to represent the simple

[7] This table (again modulo changes in the examples) is from Blackburn (1990, 1994).

past sentence *Hans ran*, asserts that there is a reference time in the past (picked out by i) and that the running took place *then*.

Now consider the future perfect. As Comrie demands, we have given this tense a unique representation—namely $F(i \wedge P\varphi)$—and it is not difficult to see that this representation correctly covers all three possibilities diagrammed by Reichenbach. Assume we are working in a model $\mathfrak{M} = (T, \prec, V)$. Let s (the speech time), r (the reference time) and e (the time of event) be any three elements of T such that $s \prec r$ and $e \prec r$; note that we have left the relation between s and e unspecified. Now, suppose that $V(i) = \{r\}$; that is, let's use the nominal i to pick out the reference time. Further suppose that $\mathfrak{M}, e \models \varphi$; that is, assume that the event of interest, represented by φ, is indeed taking place at the event time. Then, utterly straightforwardly, we have that $\mathfrak{M}, s \models F(i \wedge P\varphi)$. Reichenbach's diagrams explicitly display the three possible configurations of speech, reference and event times that make sentences in the future perfect true; the hybrid representation $F(i \wedge P\varphi)$ encapsulates what they have in common. Furthermore, note that we don't need to make additional assumptions about $\prec$ to establish this. Reichenbach's diagrams might be taken to suggest that assumptions about the transitivity or linearity play some role here; the simple model-theoretic argument just given shows that they are not.

4 Iterating Temporal Reference

So Reichenbach's analysis of tense fits well with hybrid tense logic. Moreover, there is a natural generalization of Reichenbach's ideas—one that Comrie argues leads to a general definition of tense in natural language—that can be captured in much the same way. Intriguingly, in *PPF* Prior sees this generalization quite clearly. He is first led to it via an example showing that Reichenbach's system is "too simple". This in turn suggests to him a neat observation concerning reference points which suggests that Reichenbach's system is "too complicated". Let's look closer at the example and the simplification, and then tie them together in hybrid logic.

As Prior observes (1967, p. 13), certain sentences seem to require more than one point of reference. Prior gives an example, namely *I shall have been going to see John*. This indeed seems to require two points of reference, namely R1 and R2. It's harder to say exactly how this sentence should be read, though arguably the pragmatically most plausible pattern they will exhibit is S–R2–E–R1.[8] Be that as it may, it is clear that the following hybrid formula covers all the semantic possibilities:

$$F(i \wedge P(j \wedge F(\textit{I see John}))).$$

Here the nominal i picks out the reference time R1 and j picks out reference time R2. Now consider Prior's remark on this example:

> But once this possibility is seen, it becomes unnecessary and misleading to make such a sharp distinction between the point or points of reference and the point of speech; the point of speech is just the *first* point of reference. [...]

[8] This is what Prior claims, though he notes that other readings are possible as well (Prior 1967, p. 13). Another might be S–R2–R1–E.

> This makes pastness and futurity *always* relative to *some* point of reference—maybe the first one (i.e. the point of speech) or maybe some other. (Prior 1967, p. 13)

Prior argues here very much as a logician. As he says, Reichenbach has missed a simplification: speech time is just another reference time.[9] Furthermore, Reichenbach has missed a generalization: we should be able to iterate the use of reference points indefinitely. But Prior is also missing something, or rather failing to take something sufficiently seriously. Prior seems to argue that Reichenbach's points of reference are simply *the points that are used when evaluating embedded tense operators*. For example, if we evaluate a tense logical formula $FP\varphi$ we first need to look forward to some future time, and then look back into the past from it to check the truth of φ. The intermediate times we encounter in the course of such formula evaluations are (Prior seems to say) the appropriate logical generalization of Reichenbach's points of reference. As we read him, his critique of Reichenbach in *PPF* is that ordinary tense logic, and indeed, even Findlay's pioneering work, already encompasses and extends Reichenbach's system.[10]

But viewed from the perspective of contemporary natural language semantics, this is naive; Prior has greatly underestimated how central temporal reference is. Moreover, temporal reference must be *explicit*. At the end of the paper we will give a simple three sentence narrative displaying obvious anaphoric dependencies between the three reference times. To capture the semantics of such dependencies, we must have some way of referring to reference times and expressing the required relationships between them. The tools of hybrid logic give us what we need to handle such cases simply. Prior's notion of 'implicit' reference points that arise as a side effect of operator iteration are simply not helpful here.

But to return to the generalization: Comrie in (1981) and (1985, pp. 122-30) makes an analogous point, and independently develops much the same generalisation; but Comrie's approach is driven by the demands of linguistic data rather than logical generality. Moreover, as a linguist, he is under no illusions as to the importance of temporal reference—his main concern is to find a suitable way of iterating explicit temporal reference.

In short, Prior sees the need for iterated representations, but does not see that they must be representations that allow explicit temporal reference. Comrie sees the importance of temporal reference, and looks for an appropriate mechanism for generating it. And the point we wish to make here is this: hybrid logic allows us to do both. Simply iterate hybrid logical representations as follows. Define a *primitive tensed form* to be any hybrid tense logical instance of $P\varphi$, φ, or $F\varphi$, where φ contains no

[9] This is an interesting observation: indeed (Binnick 1991), a standard text on tense and aspect, has four index entries on it, a rare example in the linguistic literature of Prior's work being viewed as of more than historic interest. As Binnick points out, similar ideas were expressed in (Allen 1966), and he calls this the Prior/Allen concept.

[10] For another account of the relationship between the work of Reichenbach and Prior (one couched in the language of orthodox Priorean tense logic rather than hybrid logic) we refer the reader to Section 2.4 of (Øhrstrøm and Hasle 1995). The authors argue that the earlier ideas of the Danish linguist Otto Jespersen (see (Jespersen 1965)) which inspired Reichenbach, fit better with orthodox tense logic than Reichenbach's own work. They also show that Prior's own brief remarks on Jespersen are somewhat oversimplified.

tense operators; so primitive tensed forms are just (boolean combinations of) basic information, prefixed by at most one tense operator. Then stipulate that if ψ is a tensed form then so are

$$P(r \wedge \psi) \text{ and } F(r \wedge \psi),$$

where r is a meta-variable over nominals. This recursive schema generates all the tensed forms used in Figure 2, the $F(i \wedge P(j \wedge F\varphi))$ used above to represent *I shall have been going to see John*, and infinitely many others. Moreover, according to Comrie, most tense forms in natural language follow this iterative pattern. And as we shall now see, this pattern points towards a further generalization.

5 More Propositional Restrictors

Consider a complex tensed form, say $P(r \wedge \psi)$. This analyses a tense into two components, a shift (carried out by the tense operator) and a restriction. Now, as we have just defined them, only one type of restriction is possible, namely that φ be true at the particular point in the past named by a nominal. But nothing prevents us from reinterpreting the meta-variable r in our recursive definition from ranging over a richer class of (propositional) restrictors. The two most obvious are various kinds of adverbials (such as dates and times) and indexical terms (such as *now*, *yesterday*, *today* and *tomorrow*). Such items can be represented as special propositional symbols, and we will now give some simple examples.[11]

Consider the sentence *John died in 1979*. We can represent this by:

$$P(\textit{1979} \wedge \textit{John die}).$$

That is, the restrictor slot has been filled by the propositional symbol *1979* which is true at all and only the points in the unbroken interval of time 1979. So a past-tensed formula is true if there is a time in the past at which John died, and this point lies within the year 1979. That is, the special propositional symbol acts as restrictor on the range of past times relevant to the truth of the matrix expression. The more complex sentence *John died at 10.42 on the 11th of June 1979* can be handled similarly:

$$P(\textit{10.42} \wedge \textit{11th} \wedge \textit{June} \wedge \textit{1979} \wedge \textit{John die}).$$

Indexicals pose more substantial problems. For example, we would like a semantics in which the sentence *John will die yesterday* will always be *false*, as the forward looking future tense conflicts semantically with the indexical *yesterday* which refers to the preceding day. But as was mentioned in the previous footnote, such indexicals have been successfully modeled within hybrid logic, and the natural shift-and-restrict representation for this sentence, namely

$$F(\textit{yesterday} \wedge \textit{John die})$$

[11] Special propositional symbols used in this way were introduced in Blackburn (1990, 1994); this work covers most common time and calendar terms plus the four indexicals mentioned above. The approach has been logically explored recently in a more general (and more elegant) semantic setup that makes use of Kaplan-style character functions; we refer the reader to Blackburn and Jørgensen (2012, 2013).

is guaranteed to be false; see Blackburn and Jørgensen (2012, 2013) for details.[12]

6 Concluding Remarks

In this paper we have combined the ideas of Prior and Reichenbach in a single framework, hybrid tense logic. We have done so in a way that overcomes what many linguists regard as the main defect of Reichenbach's original schema, namely that it gives multiple representations to sentences in the future perfect and the future-in-the-past. Moreover, we have shown that the approach can be generalized to an iterative schema that allows for multiple points of reference, a possibility noted by Prior, and demanded by Comrie. We then sketched how this schema can be further generalized to a shift-and-restrict paradigm in which special propositional symbols (for adverbials and indexicals) act as restrictors on the range of the tense operators. We believe that our iterative schema captures the core of Comrie's ideas rather elegantly, but in the case of Prior, matters are less clear. Prior did not attach great weight to temporal reference, and we read Prior as believing (in our view, mistakenly) that everything of real importance in Reichenbach is automatically covered (and indeed generalized) by the iterability of operators in ordinary tense logic.

Why did Prior not attach more weight to temporal reference? It may be connected with his dislike of instants, but there is another possibility. Until the 1980s, logical approaches to natural language semantics were largely restricted to the sentential level. Only with the advent of DRT, created by Hans Kamp in the early 1980s did focus shift to the semantics of discourse, and the importance of temporal reference only becomes fully apparent in the setting of multi-sentence texts.[13]

Consider the following narrative: *Vincent woke up. Something felt very wrong. Vincent reached under his pillow for his Uzi.* The states described by the first two sentences hold at the same time. The event described by the third takes place a little later. It is straightforward in hybrid logic to assert the identity of the reference times required by the first two sentences, and to capture the move forward in time required by the third:

$$\begin{aligned} & P(i \wedge \text{vincent-wake-up}) \\ \wedge\ & P(j \wedge \text{something-feel-very-wrong}) \wedge @_j i \\ \wedge\ & P(k \wedge \text{vincent-reach-under-pillow-for-uzi}) \wedge @_k P j \end{aligned}$$

[12] As a technical aside, we remark that the extensions just mentioned can often be made without losing either decidability (or the finite model property) and indeed, without raising the underlying computational complexity above that of basic hybrid tense logic. For example, if we have the "true at all times" operator $A\varphi$ at our disposal we can impose the semantic constraints required of the indexicals *yesterday*, *today*, and *tomorrow*, within the hybrid object language itself (see Blackburn and Jørgensen (2012) for details). But over linear time flows (as Prior himself was aware) $A\varphi$ is simply shorthand for $H\varphi \wedge \varphi \wedge G\varphi$, hence over linear time the addition of these indexicals costs nothing. Moreover, basic hybrid tense logic (at least over the kinds of linear and branching time flows typically encountered in natural language semantics) tends to be computationally inexpensive, and indeed often has the same complexity as the underlying tense logic (for example, over linear flows); see Areces et al. (2014) for some general results.

[13] Discourse Representation Theory (see (Kamp and Rohrer 1983; Kamp and Reyle 1993)) is often thought to have been motivated by problems involving pronominal anaphora, but as Hans Kamp has repeatedly emphasized, what originally promoted its invention were problems involving the semantics of tense in text; see (Kamp 1981). Another early paper on temporal anaphora is (Partee 1984).

Note that we have put the satisfaction operators to work for the first time: $@_j i$ asserts that the reference time i is true at the time named by the reference time j (that is, it is an identity statement) while $@_k P j$ asserts that the reference time j is true at a time lying in the past of reference time k (so it is a statement about temporal precedence). This is a simple example, but it should make the key point clear: reference times play an important anaphoric role in discourse, and hybrid logical machinery is a flexible tool for dealing with it.

And that is the note on which we would like to end: the time seems ripe to develop richer hybrid logics for modeling tense (and indeed, aspect) in natural language. Higher-order versions of hybrid logic now exist (see Areces et al. (2014)) opening the door to compositional semantic construction, and other topics (notably indexicality and hybrid inference) are well understood. Linguists have long been aware that Reichenbach's system had flaws that need fixing, and that various generalizations were needed, but they were unaware that there was a simple logical formalism—hybrid tense logic—that fixed these flaws and offered new tools for temporal reference. But they can hardly be blamed for this; its inventor was unaware of it too.

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